

RANDALL-SUNDRUM VS. HOLOGRAPHIC BRANEWORLD

NEVEN BILIC

*Division of Theoretical Physics, Rudjer Bošković Institute, Bijenička 54,
Zagreb, Croatia*



A mapping between two braneworld cosmologies – Randall-Sundrum and holographic – is explicitly constructed. The cosmologies are governed by the appropriate modified Friedman equations. A relationship between the corresponding Hubble rates is established.

1 Introduction

Braneworld cosmology is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk. We will consider two types of braneworlds in an AdS_5 bulk. In a holographic braneworld universe a 3-brane is located at the boundary of the asymptotic AdS_5 . The cosmology is governed by matter on the brane in addition to the boundary CFT. In the second Randall-Sundrum (RSII) model¹ a 3-brane is located at a finite distance from the boundary of AdS_5 . The model was originally proposed as an alternative to compactification of extra dimensions.

A cosmology on the brane is obtained by allowing the brane to move in the bulk along the fifth dimension z . Equivalently, the brane is kept fixed at $z = z_{\text{br}}$ while making the metric in the bulk time dependent. The time dependent bulk spacetime with line element

$$ds_{(5)}^2 = \frac{\ell^2}{z^2} (n^2(\tau, z)d\tau^2 - a^2(\tau, z)d\Omega_\kappa^2 - dz^2), \quad (1)$$

may be regarded as a z foliation of the bulk with an FRW cosmology on each z slice. In particular, at $z = z_{\text{br}}$ we have the RSII cosmology and, at $z=0$, the holographic cosmology. The Friedmann equation on the brane is modified

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho + \left(\frac{4\pi G_N \ell}{3}\right)\rho^2 + \frac{\mu\ell^2}{a^4}, \quad (2)$$

where ℓ is the AdS_5 curvature radius, $H = \dot{a}/(na)$ is the Hubble rate and μ is the parameter related to the bulk black-hole mass $\mu = (8G_5 M_{\text{bh}})/(3\pi\ell^2)$.

In the RSII model by introducing the boundary in AdS_5 at $z = z_{\text{br}}$ instead of $z = 0$, the model is conjectured to be dual to a cutoff CFT coupled to gravity, with $z = z_{\text{br}}$ providing the

IR cutoff. This conjecture then reduces to the standard AdS/CFT duality as the boundary is pushed off to $z = 0$. This connection involves a single CFT at the boundary of a single patch of AdS_5 . In the original RSII model one assumes the Z_2 symmetry $z \leftrightarrow z_{\text{br}}^2/z$, so the region $0 < z \leq z_{\text{br}}$ is identified with $z_{\text{br}} \leq z < \infty$, with the observer brane at the fixed point $z = z_{\text{br}}$. Hence, the braneworld is sitting between two patches of AdS_5 , one on either side, and is therefore dubbed “two sided”. In contrast, in the “one-sided” RSII model the region $0 \leq z \leq z_{\text{br}}$ is simply cut off so the bulk is the section of spacetime $z_{\text{br}} \leq z < \infty$.

The variation of the action yields Einstein’s equations on the boundary²

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_N(\gamma\langle T_{\mu\nu}^{\text{CFT}} \rangle + T_{\mu\nu}^{\text{matt}}), \quad (3)$$

where $T_{\mu\nu}^{\text{matt}}$ is the energy-momentum tensor associated with matter on the holographic brane and $T_{\mu\nu}^{\text{CFT}}$ the energy-momentum tensor of the CFT on the boundary. The parameter γ takes on the value 1 and 2 for the 1-sided and 2-sided RSII model, respectively. According to the AdS/CFT prescription, the expectation value $\langle T_{\mu\nu}^{\text{CFT}} \rangle$ is obtained by functionally differentiating the renormalized on-shell bulk gravitational action with respect to the boundary metric³. From (3) one derives the Friedmann equation at the boundary

$$\mathcal{H}_0^2 = \frac{\ell^2}{4} \left(\mathcal{H}_0^4 + \frac{4\mu}{a_0^4} \right) + \frac{8\pi G_N}{3} \rho_0. \quad (4)$$

where $\mathcal{H}_0^2 = H_0^2 + \kappa/a_0^2$ and $H_0 = \dot{a}_0/a_0$ is the Hubble rate at the holographic boundary.

A map between z -cosmology and $z = 0$ -cosmology can be constructed using⁴

$$a^2 = a_0^2 \left[\left(1 - \frac{\mathcal{H}_0^2 z^2}{4} \right)^2 + \frac{1}{4} \frac{\mu z^4}{a_0^4} \right], \quad n = \frac{\dot{a}}{\dot{a}_0}. \quad (5)$$

The Hubble rates are related by

$$\mathcal{H} \equiv H^2 + \frac{\kappa}{a^2} = \mathcal{H}_0 \frac{a_0}{a}. \quad (6)$$

Using this and (5) we can find a relation between the cosmological scales a_{br} on the brane at $z = z_{\text{br}}$ and a at on an arbitrary z -slice. First, we can express the first equation in (5) as an equation for a_0^2 , a^2 , and \mathcal{H}^2 , and similarly as another equation for a_0^2 , a_{br} , and $\mathcal{H}_{\text{br}}^2$ $z = z_{\text{br}}$. By eliminating a_0^2 from these two equations we find

$$a = \frac{a_{\text{br}}}{\sqrt{2}} \left[\left(1 + \frac{1}{2} \mathcal{H}_{\text{br}}^2 z_{\text{br}}^2 \right) \left(1 + \frac{z^4}{z_{\text{br}}^4} \right) - \mathcal{H}_{\text{br}}^2 z^2 + \mathcal{E}(z) \sqrt{1 + \mathcal{H}_{\text{br}}^2 z_{\text{br}}^2 - \frac{\mu z_{\text{br}}^4}{a_{\text{br}}^4}} \left(1 - \frac{z^4}{z_{\text{br}}^4} \right) \right]^{1/2}, \quad (7)$$

where we have introduced a two-valued step function

$$\mathcal{E}(z) = \begin{cases} +1, & \text{for } z \geq z_{\text{br}}, \\ -1, & \text{for } z < z_{\text{br}}, \text{ two-sided version,} \\ +1 \text{ or } -1, & \text{for } z < z_{\text{br}}, \text{ one-sided version.} \end{cases} \quad (8)$$

The map is schematically illustrated as

$$\begin{array}{ccc} d\tau^2 - a_0^2 d\Omega_\kappa^2 & \xrightarrow{\tau \rightarrow \tilde{\tau}} & (1/n^2) d\tilde{\tau}^2 - a_0^2 d\Omega_\kappa^2 \\ \downarrow z & & \downarrow z \\ n^2 d\tau^2 - a^2 d\Omega_\kappa^2 & \xrightarrow{\tau \rightarrow \tilde{\tau}} & d\tilde{\tau}^2 - a^2 d\Omega_\kappa^2 \end{array}$$

where τ and $\tilde{\tau}$ are the holographic and RSII synchronous times, respectively. By making use of (6) and (7) we express the Hubble rate at $z = 0$ in terms of the Hubble rate at $z = z_{\text{br}}$

$$\mathcal{H}_0^2 = 2\mathcal{H}_{\text{br}}^2 \left(1 + \frac{\mathcal{H}_{\text{br}}^2 z_{\text{br}}^2}{2} + \mathcal{E}_0 \sqrt{1 + \mathcal{H}_{\text{br}}^2 z_{\text{br}}^2 - \frac{\mu z_{\text{br}}^4}{\mathcal{A}_{\text{br}}^4}} \right)^{-1}, \quad (9)$$

where $\mathcal{E}_0 \equiv \mathcal{E}(0) = -1$ for the two-sided and $\mathcal{E}_0 = +1$ or -1 for the one-sided version of the RSII model. There is a clear distinction between the holographic maps involving 1-sided and 2-sided versions of the RSII model². In the 2-sided map the low-density regime on the RSII brane corresponds to the high negative energy density on the holographic brane. The low density regime can be made simultaneous only in the 1-sided RSII.

It is conceivable that we live in a braneworld with emergent cosmology. That is, dark energy and dark matter could be emergent phenomena induced by what happens on the primary braneworld. For example, suppose our universe is a one-sided RSII braneworld the cosmology of which is emergent in parallel with the primary holographic cosmology. If ρ_0 describes matter with the equation of state satisfying $3p_0 + \rho_0 > 0$, as for, e.g., CDM, we will have an asymptotically de Sitter universe on the RSII brane. If we choose ℓ so that the cosmological constant Λ fits the observed value, the quadratic term will be comparable with the linear term today but will strongly dominate in the past and hence will spoil the standard cosmology. However, the standard Λ CDM cosmology could be recovered by including a negative constant term in ρ_0 and fine tune it to cancel Λ up to a small phenomenologically acceptable contribution.

Acknowledgments

This work has been supported by the Croatian Science Foundation, Project No. IP-2014-09-9582.

References

1. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
2. N. Bilic, *Phys. Rev. D* **93**, 066010 (2016)
3. S. de Haro, S. N. Solodukhin and K. Skenderis, *Commun. Math. Phys.* **217**, 595 (2001))
4. P. S. Apostolopoulos, G. Siopsis and N. Tetradis, *Phys. Rev. Lett.* **102**, 151301 (2009)